Brief paper

Cooperative control for target-capturing task based on a cyclic pursuit strategy

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Received 31 August 2006; received in revised form 27 December 2006; accepted 21 January 2007
Available online 19 June 2007

Abstract

This paper studies a methodology for group coordination and cooperative control of n agents to achieve a target-capturing task in 3D space. The proposed approach is based on a cyclic pursuit strategy, where agent \( i \) simply pursues agent \( i + 1 \mod n \). The distinctive features of the proposed method are as follows. First, no communication mechanism between agents is necessary and thus it is inherently a distributed control strategy. Also, it is a simple robust memoryless control scheme which has self-stability property. Finally, it guarantees a global convergence of all agents to the desired formation. Further, it is also guaranteed that no collision occurs. Simulation examples are given to illustrate the efficacy of the proposed method and the achievement of a desired pursuit pattern in 3D space.

Keywords: Multi-agent systems; Cooperative robot systems; Distributed control; Pursuit problems; Circulant matrices

1. Introduction

Formation control which coordinates the motion of relatively simple and inexpensive multiple agents is one of the essential technologies that enable agents to cover a larger operational area and achieve complex tasks (see e.g., Pettersen, Gravdahl, & Nijmeijer, 2006). Recently, several research groups developed coordination control strategies which achieve an enclosing formation around a specific area (object) by multiple agents using local information (see Kobayashi, Otsuno, & Hosoe, 2006; Marshall, Broucke, & Francis, 2004, 2006; Sepulchre, Paley, & Leonard, 2006; Sinha & Ghose, 2005, and the references therein). This type of coordination of motions is not only interesting but also significant, because it has many potential applications from an engineering standpoint as mentioned in Marshall et al. (2004, 2006) and Sepulchre et al. (2006). For instance, it is useful when hazardous terrestrial/oceanographic exploration, military surveillance and rescue operation are performed by cooperative multi-agent systems.

In this line of research, Kobayashi et al. (2006) proposed a control scheme for multiple agents for target-capturing behavior. In their paper, \( n \) identical autonomous mobile robots moving on a 2D surface have been considered. They developed a decentralized control law based on a gradient descent method, where each agent’s behavior is determined by using some local information about the target object and two other agents in its neighborhood. Their method guarantees that agents’ coordination finally results in a circular formation enclosing the target object which moves freely in the plane. However, if a similar global formation could be obtained with less information, it would be more attractive in terms of simplicity of control law and implementability.

On the other hand, the formation control strategies for multi-agent systems under cyclic pursuit have recently been investigated (see e.g., Lin, Broucke, & Francis, 2004; Marshall et al., 2004, 2006; Sinha & Ghose, 2005 and the references therein). From an engineering standpoint, the cyclic pursuit strategy may become one of the attractive control methods, since it requires minimum number of communication links, and inherently does not employ a leader agent. Among such approaches, Marshall et al. (2004, 2006) proposed the formation control...
under cyclic pursuit for multiple agents with motion constraints moving in a plane. They showed that the agents finally can assemble in a circular formation under certain conditions, which is similar to that of Kobayashi et al. (2006). However, it seems to be difficult in their method to have multiple agents to enclose the freely-moving target object. Furthermore, in order to make it applicable to real world problems such as space/submarine surveyming, the development of control scheme for multi-agent systems in 3D space is required.

This paper proposes a distributed cooperative control method based on a cyclic pursuit strategy in a target-capturing task by multi-agent systems. First, we consider a group of \( n \) agents dispersed in 3D space. Then, a distributed controller is developed based on a modified cyclic pursuit methodology. This control scheme guarantees the achievement of the desired global behaviors of agents by using only a simple and local information; each agent individually decides its behavior based on local information about only one other agent and the target object. Thus, it alleviates the information requirements that used in Kobayashi et al. (2006). Note that the agent dynamics are not considered and full actuation is assumed in this paper.

The rest of the paper is organized as follows. Section 2 describes the multi-agent systems and our control objective. Section 3 presents a formation control scheme to achieve target capturing by multiple agents in 3D space. Section 4 presents numerical examples to demonstrate the effectiveness of the proposed method. Finally, concluding remarks are given in Section 5.

2. System description and control objective

Consider the group of \( n \) agents dispersed in 3D space as shown in Fig. 1. Each agent is modeled as an autonomous point mass and all agents are ordered from 1 to \( n \); i.e., \( P_1, P_2, \ldots, P_n \). Denote the position vectors of the target object and agent \( P_i \) (\( i = 1, 2, \ldots, n \)) in the inertial frame by \( p_{\text{obj}}(t) \in \mathbb{R}^3 \) and \( p_i(t) \in \mathbb{R}^3 \), respectively. Suppose that each agent is described as

\[
\dot{p}_i = u_i ,
\]

where \( u_i \in \mathbb{R}^3 \) is the control input. It is assumed that agent \( P_i \) can measure the following vectors:

\[
\mathbf{r}_i := p_i - p_{\text{obj}}, \quad \mathbf{a}_i := p_i - p_{i+1} .
\]

Define the target-fixed frame \( \{ \mathbf{X}_{\text{obj}} \} \) where the origin is at the center of target object, and \( X_{\text{obj}}, Y_{\text{obj}}, Z_{\text{obj}} \)-axes are parallel to \( x-, y-, \) and \( z- \) axes of the inertial frame, respectively. Let \( d_i \) denote the projected vector of \( r_i \) onto the \( X_{\text{obj}}-Y_{\text{obj}} \) plane in the target-fixed frame, and define the following scalars:

\[
\theta_i = \angle(e_i, d_i), \quad z_i = \angle(d_i, r_i), \quad r_i := |r_i| ,
\]

where \( e_i \) denotes the unit vector in the \( X_{\text{obj}} \)-direction of \( \{ \mathbf{X}_{\text{obj}} \} \) and \( \angle(x, y) \) denotes the counterclockwise angle from the vector \( x \) to the vector \( y \). Then, \( r_i \) can be represented as

\[
r_i = [r_i \cos \theta_i \cos z_i, r_i \sin \theta_i \cos z_i, r_i \sin z_i]^T.
\]

Note that since \( r_{i+1} = r_i - a_i \) and \( \delta \theta_i := \theta_{i+1} - \theta_i \) can be calculated in a similar way based on (2). Let \( R \) denote the required distance between the target object and the agents. We define \( P_{i+1} \) as prey agent of \( P_i \).

Now, we consider how to form a geometric pattern for the target-capturing task by the group of \( n \) agents. The detailed control objectives are stated as follows (see Fig. 2):

- (A1) \( n \) agents enclose the target object at uniformly spaced angle and maintain this angle.
- (A2) Each agent approaches to the target object and finally keeps the distance \( R \).
- (A3) The angle \( z_i \) which corresponds to the altitude of each agent converges to the desired one \( \Phi \).

Note that for the sake of clarity and page limitation, this paper only considers the equal convergence positions for all agents; i.e., \( R_1 = R_2 = \cdots = R_n = R \) and \( \Phi_1 = \Phi_2 = \cdots = \Phi_n = \Phi \), while the distinct ones for each agent can be assigned.

In the following section, we will derive the control law which achieves the above objectives (A1)–(A3).
Section 2. can realize the required geometric formation mentioned in the cyclic pursuit strategy for target-capturing task, which proposes a distributed cooperative control scheme motivated using local information. As one of the feasible methods, we desired global behavior through relatively simple control laws.

3. Formation control based on a cyclic pursuit

From the practical viewpoint, it is important to achieve the desired global behavior through relatively simple control laws using local information. As one of the feasible methods, we propose a distributed cooperative control scheme motivated by the cyclic pursuit strategy for target-capturing task, which can realize the required geometric formation mentioned in Section 2.

In order to develop such control strategy, it is first assumed that $n$ agents are dispersed in a counterclockwise star formation (Lin et al., 2004) at the initial time instant as shown in Fig. 3, where $d_i > 0$, $0 < \delta \theta_i < 2\pi$ for $i = 1, 2, \ldots, n$, and $\sum_{i=1}^{n} \delta \theta_i = 2\pi$. Note that a clockwise formation can be treated in a similar way. Then, the proposed local control law for the $i$th agent $P_i$ is described as

\[
\dot{\theta}_i(t) = k_1 \delta \theta_i(t), \quad (5)
\]

\[
\dot{r}_i(t) = k_2(R - r_i(t)), \quad (6)
\]

\[
\dot{z}_i(t) = k_3(\Phi - z_i(t)), \quad (7)
\]

where

\[
\delta \theta_i(t) := \theta_{i+1}(t) - \theta_i(t), \quad i = 1, 2, \ldots, n - 1,
\]

\[
\delta \theta_n(t) := \theta_1(t) - \theta_n(t) + 2\pi, \quad i = n
\]

and $k_1$, $k_2$, and $k_3$ ($> 0$) are controller gains, which are the design parameters.

In order to analyse the overall multi-agent system, we will rewrite (5) in the following vector form:

\[
\dot{\theta}(t) = A\theta(t) + B,
\]

with

\[
A := \text{circ}(-k_1, k_1, 0, \ldots, 0) \in \mathbb{R}^{n \times n},
\]

\[
B := [0, 0, \ldots, 0, 2k_1\pi]^T \in \mathbb{R}^n,
\]

where $\theta := [\theta_1, \theta_2, \ldots, \theta_n]^T \in \mathbb{R}^n$ and circ denotes the circulant matrix. Next, some basic results for circulant matrices will be presented (for details, refer to Marshall, 2005).

**Lemma 1.** Every circulant matrix $C \in \mathbb{R}^{n \times n}$ can be represented by

\[
C = \text{circ}(c_1, c_2, \ldots, c_n)
= c_1 I_n + c_2 \Pi_n + c_3 \Pi_n^2 + \cdots + c_n \Pi_n^{n-1},
\]

where $\Pi_n := \text{circ}(0, 1, 0, \ldots, 0) \in \mathbb{R}^{n \times n}$. Further, the circulant’s representer is defined as

\[
p_C(\lambda) = c_1 + c_2 \lambda + c_3 \lambda^2 + \cdots + c_n \lambda^{n-1},
\]

since $C = p_C(\Pi_n)$. Then, the eigenvalues of $C$ are $\lambda_i = p_C(\omega^{i-1})$, where $\omega := e^{2\pi i/n}$ with $j = \sqrt{-1}$ and $i = 1, 2, \ldots, n$.

The control objectives (A1)–(A3) in Section 2 can be formulated explicitly as follows:

(A1') $\delta \theta_i(t) \rightarrow 2\pi/n$ (rad) as $t \rightarrow \infty$,

(A2') $r_i(t) \rightarrow R$ as $t \rightarrow \infty$,

(A3') $z_i(t) \rightarrow \Phi$ (rad) as $t \rightarrow \infty$,

for $i = 1, 2, \ldots, n$.

Then, the main result of the paper is stated as follows:

**Theorem 1.** Consider the system of $n$ agents. It is assumed that all agents are initially arranged in a counterclockwise formation as shown in Fig. 3, where $d_i(0) > 0$, $0 < \delta \theta_i(0) < 2\pi$ ($i = 1, 2, \ldots, n$) and $\sum_{i=1}^{n} \delta \theta_i(0) = 2\pi$. Then, the control laws (5)–(7) achieve (A1')–(A3') simultaneously. Also, under the control laws (5)–(7), all agents remain in a counterclockwise formation. Further, no collision occurs during the arrangement of agents evolves.
Proof. Since it is obvious that (6) implies (A2’) and (7) implies (A3’), we will prove that (A1’) is achieved based on a similar idea used in Lin et al. (2004). It holds from (9) that \( \hat{\theta}(t) = A\hat{\theta}(t) \). Since \( A \) is a circulant matrix, its representor is \( p_A(\lambda) = k_1(1 + \lambda) \) and the eigenvalues are given as \( \lambda_i = p_A(e^{i\omega}) \), \( i = 1, 2, \ldots, n \), from Lemma 1. Alternatively, the eigenvalues can be rewritten in complex form as

\[
\lambda_i = k_1 \left[ \cos \left( \frac{2\pi(i - 1)}{n} \right) - 1 \right] + jk_1 \sin \left( \frac{2\pi(i - 1)}{n} \right),
\]

where \( i = 1, 2, \ldots, n \). Since \( k_1 > 0 \), \( A \) always has exactly one zero eigenvalue, \( \lambda_1 \), while the remaining \( n - 1 \) eigenvalues \( \lambda_i \), \( i = 2, 3, \ldots, n \), lie strictly in the left-half complex plane. It means that \( \hat{\theta} \) converges to the null space \( \{ \sigma | \sigma = \sigma I_n, \sigma \in \mathbb{R} \} \), \( I_n = [1, 1, \ldots, 1]^T \in \mathbb{R}^n \), which corresponds to \( \lambda_1 = 0 \); i.e., \( \hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_n]^T \) satisfies \( \hat{\theta}_1 = \hat{\theta}_2 = \cdots = \hat{\theta}_n = \sigma \) in the steady state. It shows that the angular velocities \( \hat{\theta}_i \) of all agents around \( Z_{obj} \)-axis converge to the same value \( \sigma \). Moreover, due to the distinctive feature of cyclic pursuit in (9), we have

\[
\sum_{i=1}^{n} \hat{\theta}_i(t) = 2k_1\pi \quad \text{for all } t \geq 0.
\]

Therefore, it holds that

\[
\sum_{i=1}^{n} \hat{\theta}_j(t) = 2k_1\pi \quad \Rightarrow \quad \hat{\theta}_j = \sigma = 2k_1\pi/n.
\]

Further, from (5) and (14), \( \delta \hat{\theta}_i(t) = 2\pi/n \) as \( t \to \infty \), which proves (A1’).

Next, we prove that no collision occurs. It will be proved by contradiction. Suppose that some \( \delta \hat{\theta}_i \), namely \( \delta \hat{\theta}_1 \), becomes zero at time instant \( t_1 \), while \( \delta \hat{\theta}_{\ell+1}(t_1) > 0 \). Hence, we have

\[
\delta \hat{\theta}_{\ell}(t_1) = 0, \quad \delta \hat{\theta}_{\ell+1}(t_1) > 0, \quad \delta \hat{\theta}_i(t) > 0, \quad t \in [t_0, t_1), \quad i = 1, 2, \ldots, n,
\]

where \( t_0 = 0 \). That is, \( t_1 \) is the time when a collision occurs (the conditions \( r_\ell = r_{\ell+1} \) and \( x_t = x_{t+1} \) are assumed). By differentiating \( \delta \hat{\theta}_\ell \) with respect to time, one obtains

\[
\frac{d}{dt}(\delta \hat{\theta}_\ell) = \delta \hat{\theta}_{\ell+1} - \delta \hat{\theta}_\ell = k_1 \delta \hat{\theta}_{\ell+1} - k_1 \delta \hat{\theta}_\ell.
\]

Here, \( k_1 \delta \hat{\theta}_{\ell+1}(t_1) > 0 \) since \( k_1 > 0 \) and \( \delta \hat{\theta}_{\ell+1}(t_1) > 0 \) from assumption (16). Thus, we have

\[
\frac{d}{dt}(\delta \hat{\theta}_\ell) > -k_1 \delta \hat{\theta}_\ell.
\]

Since \( \delta \hat{\theta}_\ell(t) > 0 \) for \( t \in [t_0, t_1) \) from assumption (17), it holds that

\[
\delta \hat{\theta}_\ell(t) > e^{-k_1(t-t_0)} \delta \hat{\theta}_\ell(t_0) > 0, \quad t \in [t_0, t_1).
\]

It contradicts (15) in terms of the continuity of \( \delta \hat{\theta}_\ell \). Therefore, any collision cannot occur. Also, a counterclockwise formation is preserved.

The above theorem implies that the control laws (5)–(7) guarantee that all agents assemble into the desired formation around the freely-moving target object in 3D space. It is also important to note that the proposed method guarantees the collision avoidance which is crucial in multi-agent systems.

The control scheme has additional distinctive features as follows: each agent individually obtains the required information using the sensor systems implemented on its body, which means that no communication mechanism between agents is introduced. In addition, it is a memoryless controller in the sense that each agent determines the next behavior based only on the current information on its prey, independently of the past behavior of its prey. Thus, it is an easily implementable method from the engineering viewpoint. Moreover, it is a practical method, because it is robust against any finite number of transient measurement errors (e.g., the case when the prey agent is invisible from its follower for a period of time). These properties will be verified through simulation studies in the following section.

It may be in order to describe \( \dot{u}_i \) explicitly. From (1), (2) and (4), it is straightforward to obtain

\[
u_i = \dot{r}_i + \dot{p}_o.
\]

subject to (5)–(7), and

\[
\dot{r}_i = \begin{bmatrix}
\cos \theta_i \cos \varphi & -r_i \sin \varphi \cos \varphi & -r_i \cos \varphi \sin \varphi \\
r_i \sin \theta_i \cos \varphi & -r_i \sin \varphi \cos \varphi & -r_i \cos \varphi \sin \varphi \\
r_i \cos \theta_i & 0 & r_i \cos \varphi \\
\end{bmatrix}
\dot{\theta}_i.
\]

Note that it is inevitable to exploit \( \dot{p}_o \) in order to achieve target capturing without any errors irrespective of the control strategies. When each agent knows the target velocity in the steady state, (A1)–(A3) will be satisfied. We will evaluate the performance in the case where \( \dot{p}_o \) is not available at all in the next section.

4. Simulation examples

4.1. Case 1

To illustrate the dynamic performance of the proposed distributed cooperative control scheme, a simulation is carried out in which \( n = 9 \) agents are randomly dispersed in 3D space at first and finally should achieve the required formation stated in Sections 2 and 3. Specifically, the desired formation is chosen to be given by \( \delta \hat{\theta}_i = 2\pi/9 \) (rad) \( (i = 1, 2, \ldots, 9) \), \( R = 5 \) and \( \varphi = 0 \) (rad), while the parameters of the initial position are given as shown in Table 1. The controller gains \( k_1, k_2 \) and \( k_3 \) in (5)–(7) are chosen as \( k_1 = k_2 = k_3 = 5 \). The sampling time is \( t_s = 0.01 \) (s) and the simulation is performed for \( t = 5 \) (s). The path for target object is set as follows: \( p_o(t) = [3t, 0, 0]^T \) for \( t \in [0, 1.5] \); \( p_o(t) = [3t, -3(t - 1.5), 0]^T \) for \( t \in (1.5, 3] \); \( p_o(t) = [9, -4.5, 0]^T \) for \( t \in (3, 5) \). Note that each agent does not know \( \dot{p}_o(t) \) (i.e., \( \dot{p}_o(t) = 0 \) in (21)). The simulation results are shown in Figs. 4–6. First, Fig. 4 illustrates the resulting position trajectories of a group of nine during the simulation: the agents assemble into the desired configuration and track the freely-moving target object simultaneously. Fig. 5 depicts the
Table 1
Parameters of the initial position

<table>
<thead>
<tr>
<th>$P_i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_i$</td>
<td>0.175</td>
<td>0.698</td>
<td>1.222</td>
<td>1.745</td>
<td>2.269</td>
<td>2.793</td>
<td>3.491</td>
<td>4.189</td>
<td>4.712</td>
</tr>
<tr>
<td>$r_i$</td>
<td>30</td>
<td>25</td>
<td>10</td>
<td>20</td>
<td>15</td>
<td>30</td>
<td>25</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>$z_i$</td>
<td>1.484</td>
<td>1.222</td>
<td>1.396</td>
<td>1.047</td>
<td>1.484</td>
<td>1.222</td>
<td>0.698</td>
<td>1.396</td>
<td>0.960</td>
</tr>
</tbody>
</table>

Trajectories of all agents projected onto $x$–$y$ plane. They show that all agents converge to a circular formation around the target object and maintain the form of an equilateral and equiangular polygon. The changes of $\delta \theta_i$ with respect to time are plotted in Fig. 6, where all $\delta \theta_i$ finally converge to $2\pi/9$ (rad), respectively. The above simulation results clearly demonstrate that the control goals (A1′)–(A3′) mentioned in Section 3 are achieved.

4.2. Case 2

To consider a more realistic scenario, it is assumed that the prey agent is invisible from its follower for a period of time, $t \in [t_{i_1}, t_{i_2}]$, shown in Fig. 7. It means that the $i$th agent $P_i$ might be unable to take the measurement of $\delta \theta_i$ which is used in controller (5), since $P_{i+1}$ is invisible for some time $t \in [t_{i_1}, t_{i_2}]$. In this case, we can set $\delta \theta_i$ as $\delta \theta_i(t) = \delta \theta_i(t_{i_1} - t)$ where $t \in [t_{i_1}, t_{i_2}]$. Simulations are performed in the case of $n = 9$ agents with the same initial parameters and the desired formation parameters given in Case 1, except $\Phi_i$ (rad). In this case, we set $\Phi_1 = 1.047$, $\Phi_2 = 0.785$, $\Phi_3 = 0.524$, $\Phi_4 = 0.262$, $\Phi_5 = 0$, $\Phi_6 = -0.262$, $\Phi_7 = -0.524$, $\Phi_8 = -0.785$, $\Phi_9 = -1.047$. The simulation results clearly demonstrate that the control goals (A1′)–(A3′) mentioned in Section 3 are achieved.
methods. Furthermore, the developed control scheme guarantees no collision between agents with simple memoryless controllers. Simulations illustrated the efficacy of the proposed method and the achievement of a circular pursuit pattern in 3D space. Future research will be devoted to extend the proposed methodology for handling agents with dynamics and motion constraints. Also, in order to improve the implementability of the proposed scheme, the method to estimate the velocity of target object should be developed. Further, the scheme should be tested experimentally to investigate the behavior and performance in a realistic environment.

References


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